

Interview with University of Maryland Mathematics Professor Jerome Dancis
January 2, 2006

The “Math Wars” have been fought since the 1980s. The debate centers on what Mathematics content should be taught and how it should be taught. In the following interview, Dr. Jerome Dancis talks about the math wars, why mathematics education standards are so low in Maryland, and what we can do about it. Dr. Dancis is an Associate Professor Emeritus in the Mathematics Department at the University of Maryland, College Park.

Disclaimer by Jerome Dancis. The informality of this question-answer format and shortage of time produces short incomplete answers. I have emphasized interesting stories; ones that are not isolated cases. It is all true, but there was not time for the whole truth. Complete answers would be many times as long.

Q: The following quote from Education World describes the Math Wars:

On one side of the conflict are the traditionalists who claim that students should learn math by memorizing and practicing basic facts and skills. On the other side are proponents of what is often called “whole math,” who deride the old “kill and drill” methods of education, claiming that children learn best when they discover, understand, and integrate knowledge through independent exploration. ⁱ

Would you say this is an accurate depiction of the longstanding debate over math education?

A: It is a common way the Math Wars conflict is misrepresented by many advocates for the Reform Math Movement. It only mentions differences in teaching styles while not mentioning the more important differences in curriculum (much weaker under Reform Math) and the even more important differences in the amount of mathematics learned (much less under Reform Math).

In fact, mathematicians advocate the importance of both conceptual understanding (Reform) and basic skills (Traditional). They are not mutually exclusive. Basic skills are necessary for conceptual understanding and problem-solving. ⁱⁱ

In theory, the Math Reform Movement places great emphasis on conceptual understanding, but, in practice, the Math Reform conceptual understanding is at a very low level. Math reform often mistakes vocabulary knowledge for conceptual understanding. Some of the Math Reform curriculum is wrong and some of it is misleading. ⁱⁱⁱ All the pattern recognition problems on the MD pretend Algebra exam are wrong and much of the data analysis is misleading. ^{iv}

However, there is a place for the good elements of both traditional education, such as learning the usual method of multiplication and having a coherent curriculum, and a few aspects of a Math Reform education, such as group work, in an effective mathematics program.

The Math Reform Movement has added a Data Analysis strand in most grades. Yes, students should learn some Data Analysis and “Box and Whisker” Diagrams are a very useful way to represent data. But much of the Reform Data Analysis is superficial and or *misleading* and in K-5 a lot of time is allocated to learning very little Data Analysis. (See Appendix.) Elementary school students find the “modes” of sets

of numbers, boringly, year after year after year; even though the mode is often a misleading “measure of Central tendency”.

I’ve read that the reform movement was a reaction to the low achievement in mathematics in U.S. schools during the 1980s. Was math education in the 1980s more or less rigorous than math education in previous decades?

My child, in the 1980s traditional fast academic track, was taught one third less mathematics in high school than I was in the 1950s traditional standard academic track. Serious training in deductive proofs of theorems and in word problems and Solid Geometry has disappeared from the current curriculum. The math curriculum of the 80s was severely “dumbed-down.”

According to Professor Barry Simon, Chairman of the Mathematics Department at the California Institute of Technology, “the dumbing down of high school education in the United States, especially in mathematics and science, is a crime that must be laid at the doorstep of the educational establishment”.^v

Professor Simon noted “what is really important [in math education] is the exposure [of students] to clear and rigorous arguments ... [so that] they can more readily see through the faulty reasoning so often presented in the media and by politicians”.^{vi} If students received a more rigorous math education they would also have less difficulty adjusting to and understanding college courses.

Can you give me an example of how low the math standards were in the 1980s?

The absurdly low level of Arithmetic word problems in the 1980s Traditional Curriculum was exemplified by the following word problem, which was not permitted on the State of Maryland Functional Mathematics Test:

Problem: Sally buys a loaf of bread for two dollars and a gallon of milk, also for two dollars; she gives the clerk a five dollar bill. What is the change?

This problem was not included on the Maryland Functional Math Test because the thinking required to solve this problem was considered *too hard*. Passing this exam was a MD high school graduation requirement until about 2004.

To its credit, the 1980s Traditional Curriculum, emphasized (albeit over-emphasized) arithmetic as the core of K-8 mathematics and Algebra as the core of high school mathematics. To its discredit, there is much truth in its description as “kill and drill”.

The problems with the 1980s math curriculum cried out for dramatic solutions. To the rescue or pseudo-rescue (depending on one’s perspective) came the Math Reform movement, which was largely organized by colleges of education.

Is the Reform Math movement an attempt to address the ineffective math curriculum of the 1980s?

Partially, but the Reform Math curriculum, including the Maryland State Department of Education’s curriculum (MSDE), is far more underachieving, very repetitive, even more ineffective, and even more inefficient than the 1980s math curriculum.

So the reform movement’s attempt to increase the effectiveness of math education actually resulted in lower and less effective math standards?

Yes. The Maryland School Performance Assessment Program (MSPAP) exams were an example of the Reform curriculum's lowering of math standards. Former head of the Maryland State Board of Education, Robert Embry, wrote:

I realized ... how fundamentally serious some of MSPAP's deficiencies were. For example, one question had students manipulating a dozen small pieces of paper to figure out the area of a room--seemingly to avoid making a simple calculation using the traditional paper and pencil ... Perhaps no issue about the MSPAP concerns us more than the clear lack of substantive knowledge required for the test. ^{vii}

How specifically did Reform math curricula impact student achievement in the U.S.?

The Reform Math movement's marginalization of arithmetic has led to large decreases in student computational skills. For example, Fairfax County, Virginia (with median family income of \$90,000) implemented a Reform Math curriculum in 1987. On the Grade 8 Mathematics computation portion of the IOWA test, scores dropped from 77% in 1987-88 to 60% in 1996. For African-Americans scores dropped from 50% to 35% and for Hispanics scores dropped from 67% to 41%. ^{viii} This decrease in scores was not considered a failure since arithmetic was not on Fairfax County's list of priorities.

During the period when "Reform" instruction was being used in California schools, the remediation rate in mathematics (Algebra) for freshman, at the California State University System, skyrocketed from 23% in 1989 to 54% in 1998.

What was the response in California to this increase in remediation?

In December 1997, there was a backlash against the Reform movement in California. The Reform Math orders were cancelled. Free, once again, to teach real Algebra, the California high schools proceeded to do so. The result was a dramatic drop in the remediation rate in mathematics for freshmen in the California State University System, from 54% in 1998 down to 37% in 2002. For African Americans the remediation rate dropped from 80% to 64%. ^{ix}

To the rescue from the Reformers, came a third way, namely the 1999 California Framework and Standards. ^x It contains and is better than the good parts of both the Reform Math Movement and the 1980s "traditionalists" curriculum (albeit still too many topics in K-7 and still not up to the level of the 1950s academic high school curriculum).

This framework has a unique requirement, namely that K-7 Math textbooks be free of math errors; this only applies to the California edition. For this reason, the pattern recognition problems on the Maryland HSA pretend Algebra exam may *not* be included in the California edition of Math textbooks.

Why is it important for students who do not plan to pursue math or technology in college to learn real algebra?

According to civil rights activist Dr. Robert Moses, math literacy (and algebra literacy) is a civil rights issue. In his book "Radical Equations" Dr. Moses states that "the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy." ^{xi}

Verbal literacy, says Moses, is no longer enough. Algebra "now is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the Industrial Age." ^{xii}

For many students, learning Algebra is crucial to obtaining their first or second choices for a career or college major. The lack of an appropriate math background is especially problematic because math builds on itself. Students arriving in college, not fluent in Arithmetic and Algebra, often do poorly in remedial math in college and then drop out.

Finally, real algebra in high school is important because it helps develop abstract thinking and problem-solving skills. My son-in-law is a linguist and he tells me that algebra helped him understand things in linguistics. There is a certain amount of abstract thinking and problem solving involved in algebra and problem solving is useful for anything. There is very little problem solving in other high school subjects such as History and English.

If a student does well on the Maryland High School Assessment [HSA], will the student do well in college math?

Not necessarily. The state HSA not only does not provide the background for college math; it does not provide the background for even college remedial algebra (*real* Algebra I or II); and it doesn't provide the background for high school algebra II or for high school physics.

As 40 MD college Math and engineering faculty have noted: "Maryland's mathematics standards ... neglect the math skills [like arithmetic] and conceptual understanding that are essential for real algebra" and "teaching to such a low standard will increase the already high number of students taking remedial math [that is, real Algebra] in college." ^{xiii}

Unfortunately, the Maryland HSA exams pressures school systems to prepare students for the test itself, not for college. For example, a 1999 Montgomery County staff development session for Algebra I for teachers was summarized to me as: "Do not worry about the students understanding algebra -- Just be sure they can put anything on their hand calculators." ^{xiv} This is a very good strategy if the goal is just to have students pass the MD Algebra exam. This is a counterproductive strategy if the goal is to have students avoid remedial Algebra when they enter college.

According to Blair teachers and administrators

Changes in the Algebra I curriculum brought about by the new Maryland High School Assessment Tests (HSAs) and a push by [Montgomery County Public Schools] to have more middle school students take algebra have caused many students to be seriously unprepared for higher level math.

Shortly before the introduction of the HSA, [Montgomery County Public Schools] mandated changes to the Algebra I curriculum to align the course with the tested material. ... 'We don't think the material is what they need to know to be successful,' said Blair algebra lead teacher Maria Costello.

Changes in the curriculum are cited as a main cause for students' deficiencies in basic algebra, which are manifesting themselves in higher level math courses that require an understanding of concepts taught in Algebra I. "Our Algebra II students are worse than ever. Our Pre-Calculus students are worse than ever. It's falling apart as we go up the ladder," said Costello. ^{xv}

At the March 2004 meeting of the Maryland State Board of Education, University of Maryland, Baltimore County President Freeman A. Hrabowski III called Maryland's math standards "woefully inadequate" and said repeatedly that the Algebra standards were too low. ^{xvi}

The kicker in Maryland's Algebra exam is that the reading level is higher than the math level; this produces *false negatives*, that is, many students understand the required math but flunk the exam due to reading comprehension difficulties.

What about NAEP [National Assessment of Educational Progress]? Will a student who does well on NAEP be prepared for college math?

Not necessarily. Most of the questions on the Grade 12 NAEP belong in Grade 8.

Is Maryland's high school Math curriculum effective at all?

Maryland High Schools had an effective high school curriculum, at least for many students. Of course, it was not effective for the many students who did not know Arithmetic. The overuse of calculators (this century) has resulted in students being less well prepared for college math. Students will be even less well prepared, as MD High Schools replace Real Algebra by MD (HSA) Pretend Algebra.

Is any real arithmetic or algebra being taught in Maryland schools?

Arithmetic has been marginalized. The Reform curriculum has several overarching themes or topics. Every topic must be taught each year whether it is pedagogically useful or *not!* In the Math Reform curriculum these include Arithmetic, Algebra, Data Analysis, Probability, Geometry, Measurement and a few others. As a result, Arithmetic has been almost squeezed out.

One of my colleagues' sons was in third grade in Montgomery County, where they spent six weeks on multiplication starting with $2 \times 2 = 4$ and going on to how one calculates 23×37 . No one can learn multiplication in six weeks, but those other topics fill up the curriculum.

Another colleague of mine has a child in 5th grade in Montgomery County. That child was trying to learn division, but the child does not know the multiplication tables. When students get to algebra problems, if they do not know the multiplication tables, they cannot solve $7x = 49$ by dividing both sides by 7. This is not a problem because it is not required for Maryland's pretend Algebra exam.

In Potomac, Maryland it is common for parents to hire tutors at \$50 an hour to teach their children the arithmetic that has been marginalized at school. (Potomac is a very rich town, where the million dollar houses are the cheap ones.)

A friend's son was in an expensive private school in Baltimore, where they made a point of *not* teaching the multiplication tables.

Knowing that $x + 2x = 3x$ or solving $2x = 8$, or graphing $y = \frac{3}{2}x + \frac{7}{2}$ without the aid of a graphing calculator is *not* on the syllabus for the MD Algebra exam. This is why I nick-named it Maryland's pretend Algebra Exam and why Karin Chenoweth, Washington Post's Homeroom columnist, also calls it Maryland's pretend Algebra Exam. ^{xvii}

"Even one of the people responsible for the [Maryland Algebra] test, [MSDE] official Gary Heath, said, 'We would be the first to tell you it doesn't have a lot of algebra, nor was it intended to.'" ^{xviii}

Maryland schools are replacing their real Algebra I course, with MSDE's pretend Algebra syllabus.

Why are the standards for K-12 math education so low in Maryland?

Because MSDE is not willing to set appropriate standards for teacher certification.

In order to learn Real Algebra, students need to be fluent in arithmetic. But many students leave middle school not fluent in arithmetic. To achieve fluency in arithmetic, students need teachers who are fluent in arithmetic. But fluency in arithmetic, especially calculations with fractions and decimals, is not required for MSDE's endorsement as a "highly qualified" middle school Math teacher. So what to do? Option #1 is for MSDE to raise its Math standards for "highly qualified" middle and elementary school Math teachers; this would enable MSDE to raise its Math standards for students.

Option #2 is that Arithmetic is marginalized and Algebra is reinvented as "pretend" algebra, without the arithmetic background. MSDE adopted Option #2. Maryland school systems are revamping or replacing their traditional Algebra I curriculum with MSDE's pretend algebra.

"A lot of algebra courses aren't really teaching algebra," said Tom Loveless, director of the Brown Center on Education Policy at the Brookings Institution in Washington. "And schools are giving up on kids who can't do basic arithmetic, putting them in phony algebra courses to cover up the problem."^{xix}

Do public school math teachers have a strong background in mathematics?

Many teachers have very good knowledge of Mathematics. Unfortunately for our children, many others do not. We would like to assume that MSDE's teachers' certification guarantees appropriate and correct knowledge of Mathematics. This assumption is *not* correct.

A child of mine had mathematically correct answers marked wrong on Algebra I and II exams and on a Geometry exam. This occurred at Blue Ribbon schools, which received national honors from the first President Bush as "Schools of Excellence"

That a MCPS Algebra exam writer had no understanding of the concept of "variable" in Algebra was demonstrated by the two questions, reprinted in *The Washington Post* (July 10, 2001). The Post printed these two questions to expose the fact that a question on a MCPS Algebra exam was almost identical to one on the county's review sheet for the exam.

"When [a] biology teacher had to teach [a chemistry class] at Howard High School [Howard County], how to change centimeters to meters, he just told them to move the decimal two places -- rather than illustrating the concept. ... 'Forty-five minutes later, only three of them got it.'"^{xx}

Not so difficult, 236 centimeters make 2.36 meters, just like 236 cents make \$2.36. The biology teacher should have been taught this in middle school; also changing centimeters to meters does occur in Biology lab work.

At a University of Maryland seminar for Math Education graduate students, the question came up: why is it that "a negative times a negative equals a positive". A math education professor from Harvard explained it as follows: If we consider good to be positive and bad to be negative, then doing bad things to bad people is considered good. This is a cute answer, but it has *no* connection to the correct explanation, which appears in the Appendix.

You said that the Reform movement was largely organized by Colleges of *Education*. Shouldn't professors from Colleges of *Mathematics* also have a say in how math is taught in public schools?

Only if the top priority is a *good* math curriculum. It's not only that we know math but we also know what students need to know when they enter college.

College mathematics, science and social studies professors have little if any voice in determining state education policies. For example, in 2001 the Maryland Mathematics Commission released a report that included recommendations for changing math education in Maryland.^{xxi} The only professor of mathematics on the Maryland Mathematics Commission (with a Math Ph.D.) was regularly outvoted as a minority of one.

The Commission report disparages the traditional high school mathematics curriculum, which is mostly real algebra. The report advocates Reform Math courses in data analysis, statistics, probability, and discrete mathematics. However, without this disparaged traditional background, only *superficial* courses in data analysis, statistics, probability, and discrete mathematics, as well as high school physics are possible.

Why was the Maryland Mathematics Commission so set against traditional math?

You should ask its members.

Rather than rely on mainstream mathematicians, the commission sought advice from consultants like Steve Leinwand, who support a policy that fails to prepare students for college mathematics. This is a quote from Steve Leinwand:

It's time to confront those nagging doubts about continuing to teach our students computational algorithms for addition, subtraction, multiplication, and division [like 23×37]. It's time to acknowledge that teaching these skills to our students is not only unnecessary, but counterproductive and downright dangerous! It's time to banish these vestiges of yesteryear from our schools and from our tests.^{xxii}

Why does Steve Leinwand consider the teaching of standard Arithmetic to be dangerous?

Most compelling to Leinwand is the "sense of failure and the pain unnecessarily imposed on hundreds of thousand of students in the name of mastering these obsolete procedures". This results in the lowering of their self-esteem, their self-image and their self-confidence. Ergo, it's dangerous to their mental health. So to avoid this danger to these 100,000 students, Leinwand advocates not teaching the standard method for 23×37 to any students.

Who is Steve Leinwand?

He was the math leader of the Connecticut state department of education. In 1999, he was co-chair of the US Department of Education's Expert Panel on math textbooks. The panel was stacked with people who agreed with him and the textbooks that they declared to be exceptionally good were the ones that agreed with his philosophy. In actuality, several books on the list contained "serious mathematical shortcomings". So about 200 professors, mostly of mathematics, including yours truly, and four Nobel laureates signed a public letter to the U. S. Secretary of Education calling on him to "withdraw your premature recommendations".^{xxiii}

How do Maryland teachers become "Highly Qualified" under No Child Left Behind?

One way teachers may earn the “highly qualified” middle school math teacher endorsement from MSDE is by passing the Praxis II “Middle School math” content exam. Teachers may use calculators on this exam, so fluency in fractions and decimals is *not* required.

Alternatively, middle school math teachers can use (pretend) college Math courses, specifically designed for elementary school (K-5) teachers, to partially fulfill the college math requirements for MSDE’s endorsement as a “highly qualified” middle school math teacher. The level of these Math courses is so low that college students, majoring in basket weaving, cannot get credit for these courses.

How do high school and college textbooks compare?

About 15 years ago, Frances Gulick’s child had a horrendous traditional high school Algebra II book. Dr. Gulick, a lecturer in Mathematics at University of Maryland, wrote a critique of it.

Dr. Gulick compared two topics taught in both the high school book and the college finite math book used at the University of Maryland. The college book allocated full 7 and 12-pages to the topics, with explanations that one could learn from, while the high school book allocated only two and two and a half pages, with pretend explanations that one could not learn from.

Later, I learned that a main rule of American school textbooks is called “the two page spread”, that is, the entire lesson should fit on two pages. That the Math lesson be understandable is *not* a rule of American school textbooks. An exception is that the 1999 California Math Framework requires that Math textbooks be both understandable and correct; this only applies to the California edition. Singapore K-7 Math textbooks are also both understandable and correct.

“For [a biology teacher], teaching [Math] means having to rely on the textbook ... And the books don’t usually help him explain why. He couldn’t tell his ninth-graders why a negative times a negative equals a positive. In Consumer Math recently, he gave up explaining a loan payment formula and told students that if they wanted to figure it out, it was in the book.”^{xxiv}

When a biology teacher finds a math textbook’s explanations inadequate, Grade 9 students will find the explanations *doubly* inadequate.

Who decides what will be taught?

An administrator from Prince George’s County summed up how state exams shape what is taught in the public schools. This administrator cautioned the State not to include

any item in the Core Learning Goals for the high school assessments that it did not then plan to use in the test. “Forget about us teaching anything you’re not testing. All anybody cares about anymore is if the material is going to be on the test.”^{xxv}

Are there any effective high school curricula that prepare students for college math?

MD High Schools had an effective high school Math curriculum, at least for many students. Of course, it was not effective for the many students who did not know Arithmetic. The overuse of calculators (this century) has resulted in students being less well prepared for college. As MD High Schools replace Real Algebra with MD (HSA) Pretend Algebra, students will be even less well prepared.

What prompted California to drop the Reform Math curriculum?

California developed its current math curriculum after experiencing the disastrous impacts of the reform movement. Eight years of Reform Math in California public schools resulted in remedial math (Algebra) rate, at the California State College System, skyrocketing from 22% to 52% among entering students.

The drastic drop in student math achievement resulted in a backlash against the Reform movement and led to the creation California Math Framework.

Does tutoring help with the weakest students?

Good tutoring helps.

Here's another story -- from University of California at Berkeley about 30 years ago. None of the African-American students were making it through freshman calculus. Actually neither were any White children of blue collar workers or farmers. A fellow by the name of Uri Triseman attempted all types of remediation for the Black students and they flat out didn't work.

He then organized racially-integrated math workshops for at-risk students; officially these workshops were called the Math Honors Program. It was not traditional tutoring and it was not traditional remediation. As befits an honors society, the participants worked on problems that were more difficult than the regular problems. But there was support there. They would work in teams and there was a tutor there and there was extended time. The result was that the Black workshop students' average grade was a B-; better than the average grade of C for the White non-workshop students. ^{xxvi}

What can be done to improve K-12 math education in Maryland?

Appropriate standards for teacher certification together with appropriate teacher training are crucial to improving education. A recent New York Times editorial makes this point:

No matter how hard localities try, the best-designed high schools in the world will still fail unless the states and the federal government finally bite the bullet on teacher training. That means doing what it takes to remake the teacher corps, even if it means withholding federal dollars from diploma mills pretending to be colleges of education, forcing out unqualified teachers and changing the age-old practice of funneling the least-prepared teachers into the weakest schools. ^{xxvii}

I would paraphrase its second sentence as: No matter how hard localities try, the best-designed schools in the world will still fail unless the states finally bite the bullet on standards for teacher certification.

I would recommend the following changes in order to improve math education:

1. MSDE must first bite the bullet on standards for teacher certification. Especially raising Arithmetic content standards for K-5 teachers and Arithmetic and real Algebra content standards for Middle school Math teachers.
2. Institute new state certifications for combined middle school math and science teachers and for AP Calculus teachers. An AP Calculus teacher could provide math expertise and math leadership for his/her high school's math teaching team. (Same for middle school teachers and for AP teachers in other subjects.)
3. Provide serious professional development in Arithmetic and real Algebra for those K-8 teachers who would benefit from it.
4. Provide bonuses for elementary and middle school principals and vice-principals who are fluent in Arithmetic and Real Algebra. They could provide their teachers with leadership in Arithmetic

instruction and could accurately evaluate Arithmetic instruction.

Raising the math standards for math teachers will enable the state to raise math standards for students.

5. Replace the counterproductive Maryland Content Standards with the California Standards. Better yet, use the Singapore standards for K-7^{xxviii} or use a “cut-down” version of Achieve’s K-8 math curriculum standards^{xxix} (by moving its Grades 6-8 probability material and much of the Geometry to high school). Maryland is one of 18 states that joined Achieve’s “Mathematics Achievement Partnership”. Singapore textbooks were used with great success in College Gardens Elementary (Montgomery County). It would be easy to implement Singapore standards for K-7 since the extremely well written Singapore textbooks are written in English.^{xxx}
6. For High School use textbooks approved by the state of California for this Century or academic math textbooks from the 1950’s.
7. Use the PSAT Math exam for the states Math exam for Grade 8, required by NCLB. The bulk of the PSAT Math exam consists of Arithmetic and pre-Algebra questions, but the PSAT questions require a moderate amount of thinking and analysis. Teaching to the PSAT Math exam would considerably raise the level of teaching mathematical thinking and analysis.^{xxxi}
8. Replace the MD pretend Algebra exam with the Algebra I questions on the common Math placement exam of MD community colleges.
9. Require that MD Hope scholarship recipients be able to enter college, without the need to take any remedial course. This would pressure high schools to raise standards.

Appendix. Some Mathematics: Misleading Data Analysis and The Distributive Rule

Why is it that mathematicians “define” “a negative times a negative to equal a positive”?

We do it to preserve **The Distributive Rule**: $(a+b)c = ac + bc$,
Without this rule, many simple Algebraic calculations would become very complicated.

The flavor of the reasoning goes like this: It is super useful that fifteen sevens equals ten sevens plus five sevens, for example, then: $(10+5)x7 = 15x7 = 10x7 + 5x7 = 70+35 = 105$.

This is represented algebraically by the Distributive Rule: $(a+b)c = ac + bc$,
(with a, b and c replacing 10, 5 and 7 resp.) (It is also an example of factoring).

Now plugging $a = -1$, $b = 2$ and $c = -5$ into the Distributive Rule formula yields:

$$(-1+2)(-5) = (-1)(-5) + 2(-5)$$

Hence: $-5 = (1)(-5) = (-1+2)(-5) = (-1)(-5) + 2(-5) = (-1)(-5) - 10$.

Thus: $-5 = (-1)(-5) - 10$.

We add +10 to both sides to isolate “ $(-1)(-5)$ ”, which we treat as an unknown; this yields:

$$+5 = 10 - 5 = (-1)(-5)$$

Thus $(-1)(-5) = +5$; YEA!

In this way, the Distributive Rule *forces* the rule: “negative times a negative equals a positive”.^{xxxii}

There are all sorts of examples, which *favor* “negative times a negative equals a positive”. But, none of them *justify* it. Mathematicians are *not* capricious when they write definitions; they often make them in order to preserve a useful formula, such as the Distributive Rule.

Please provide examples where the Reform Data Analysis is misleading.

The extreme sensitivity of modes to small changes in data indicates that the usefulness of modes is very limited. This is demonstrated by the following problem and comment:

Problem. Find the mode of the heights of all the children in the class.

(a) Sally uses a ruler with one-inch markings to measure all the heights, rounded to the *nearest inch*. Her data is 50, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60. What is the mode according to Sally's data?

Answer: 50

(b) Then John finds a ruler with one-half inch markings, which he uses to re-measure all the heights, rounded to the *nearest half-inch*. His data is 50, 50.5, 51, 52, 53, 54, 55, 56, 57, 58, 59.5, 59.5. What is the mode according to John's data? Answer: 59.5

(Sally's and John's slightly different measurements would occur if the exact [un-rounded] heights were 50, 50.3, 51, 52, 53, 54, 55, 56, 57, 58, 59.3, 59.7.)

Comment. Is Sally's mode (50) near John's mode (59.5)? No! They are at the *opposite* ends of the numbers. The mode for the heights is close to meaningless!

Also, the mode is called a "measure of central tendency". This is misleading terminology, since there is nothing central about either Sally's mode or John's mode. The children are all growing, none is shrinking, so none of the children's heights is *tending* toward Sally's mode.

A typical Reform Math Question. For the data provided, which measure of central tendency is the best representative? Explain why you've made your choice.

But, the choice, of which statistic to use to best describe data, usually depends on *context*, that is, *on how the information will be used*. Or to paraphrase realtors, what decides which statistic to use is context, context, context. Reform Math is *incorrectly* training students to *ignore* the context. This is demonstrated by the following problem, answers and comment. (Note: "Mean" is jargon for "average".)

Problem. Determine the total cost to our company for a 4% cost of living wage increase to all employees. (Exclude the additional taxes such as social security and Medicare). At our company, the mean annual salary is \$100,000; the median annual salary is \$25,000. The mode annual salary is \$12,000; it is the minimum wage and is earned by 10 workers. Our company has 1,001 employees, with just a few officers receiving very high salaries.

Typical Reform Math Answers to the typical Reform Math Question above: 1. The median salary of \$25,000 best represents an annual salary because there were just as many salaries below it as there were above it.

2. The mean salary of \$100,000 does *not* best represent an annual salary because there are a few extreme salaries. Note: "Mean" is jargon for "average".

Comment. But the mean *can* be used to solve the problem. The median cannot be used to solve the problem. So, in sharp contrast to the Typical Reform Math Answers, for this problem, the mean is the best representative; the median is useless.

($\{\text{Average salary}\} = \{\text{Total Salaries}\} / \{\text{Number of workers}\}$; hence $\{\text{Total Salaries}\} = \{\text{Number of workers}\} \times \{\text{Average salary}\} = 1001 \times \$100,000 = \$100,100,000$. The 4% increase is 4% of $\$100,100,000 = \$40,040,000$.) The median cannot be used to solve the problem. So, in sharp contrast to

the Typical Reform Math Answers, for this problem, the mean is the best representative; the median is useless.

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- ⁱ Starr, L. (2002). Math Wars! *Education World*. http://www.education-world.com/a_curr/curr071.shtml
- ⁱⁱ Dr. H. Wu's great article, "Basic Skills Versus Conceptual Understanding – A Bogus Dichotomy in Math Education" in *The American Educator*, American Federation of Teachers, Fall 1999.
- ⁱⁱⁱ J. Dancis, "Pattern Mis-Recognition" <http://www.math.umd.edu/~jnd/Patterns.pdf>
- ^{iv} See the sections "MD Algebra sample test Items #35 and #19 -- which are mathematically incorrect ""and Mis-education in data analysis" and "Bad Habits are Hard to Break" in "Beware the MD Algebra test " on my website at <http://www.math.umd.edu/~jnd/>
- ^v Simon, B. (6 February 1998). A Plea in Defense of Euclidean Geometry: Math education: Fewer classes require proofs--more whittling away of exposure to logic and critical thinking. *Los Angeles Times*.
- ^{vi} Simon, B. (6 February 1998). A Plea in Defense of Euclidean Geometry: Math education: Fewer classes require proofs--more whittling away of exposure to logic and critical thinking. *Los Angeles Times*.
- ^{vii} Robert C. Embry, Jr's, (former head of the Maryland State Board of Education), 1966 memorandum to Nancy Grasmick, MD State Superintendent, on the web at <https://listserv.umd.edu/cgi-bin/wa?A2=ind0108&L=ctetch-1&P=424> ; it was an MSDE state secret until summer 2001.
- ^{viii} Data from Fairfax County Federation of Teachers.
- ^{ix} <http://www.asd.calstate.edu/performance/remediation.shtml>
- ^x California State Board of Education. Standards and Frameworks. <http://www.cde.ca.gov/be/st/>
- ^{xi} Dionne, Jr., E.J. (6 March 2001). Into the Math Mix. *The Washington Post*, pg. A.23.
As leader of the Mississippi Voter Rights Project, Dr. Moses was one of the ten most important civil rights activists in the 1960s. Moses has a Harvard Ph.D. in the philosophy of mathematics. For the past decade he has been organizing the Algebra Project.
- ^{xii} Dionne, Jr., E.J. (6 March 2001). Into the Math Mix. *The Washington Post*, pg. A.23.
As leader of the Mississippi Voter Rights Project, Dr. Moses was one of the ten most important civil rights activists in the 1960s. Moses has a Harvard Ph.D. in the philosophy of mathematics. For the past decade he has been organizing the Algebra Project.
- ^{xiii} College faculty "Petition to Upgrade Maryland's Mathematics Standards".
http://www.math.umd.edu/~jnd/subhome/petition_w_sign.htm
- ^{xiv} From Silver Chips, the student newspaper of Blair High school, Montgomery county, MD 2/18/2003 State and county math standards hurt student performance by Elliott Wolf, Online Staff Writer, Technical Staff
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- ^{xix} Mathews, J. (19 August 2002). Algebra = X in One School, Y in Another; Teaching Inconsistent as Standards Waver;. *The Washington Post*, p. A.01.
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- ^{xxii} Steve Leinwand, "It's Time To Abandon Computational Algorithms," *Education Week on the Web*, February 9, 1994.
- ^{xxiii} Refer to Nov. 18, 1999 *Washington Post* This cryptically written public letter may be found at <http://www.mathmaticallycorrect.com/riley.htm>, complete with links to web sites containing (not easily found) referenced documents.

^{xxiv} Perlstein, L. Right Teacher, Wrong Class; Schools Stretch Resources by Handing Staff Unfamiliar Subjects. *The Washington Post*, pg. A.01.

^{xxv} From: Robert C. Embry, Jr's, (former head of the Maryland State Board of Education), 1966 memorandum to Nancy Grasmick, MD State Superintendent, [on the web at <http://edreform.com/forum/mspap1196.htm>]; it was an MSDE state secret until summer 2001.

^{xxvi} My report at <http://www.math.umd.edu/~jnd/Treisman.txt> And my short article, "Math Workshops", Washington Post PG Extra Section, Forum March 17, 1999

^{xxvii} Concluding paragraph of "Reinventing High School", Lead editorial The New York Times. February 1, 2005

^{xxviii} It would be easy to implement the Singapore standards for K-7 since they are extremely well written and the Singapore textbooks are written in English. After two years, the principal, the teachers, the students and the parents at College Gardens Elementary were all in favor of the books.

^{xxix} (See www.achieve.org/achieve.nsf/MAP-k-8?OpenForm)

^{xxx} The website to obtain Singapore math books is: <http://singaporemath.com>
http://singaporemath.com/new_elem_math.htm#NEM%20Order)

^{xxxi} Bowler, M. (31 March 2004). A call to raise the standards; Expectations: UMBC's president urges state education officials to push schools toward academic excellence. *The Baltimore Sun*, pg. 2.B. "When opponents of the SAT, for example, tee off on its purported bias against minorities, [University of Maryland, Baltimore County President Freeman A. Hrabowski III] replies that he has been writing SAT test questions for years -- and that minorities can, and many do, ace the SAT."

^{xxxii} That this works for all pairs of negative numbers can be checked with a somewhat similar calculation. Actually, one should have first checked that $a(-b) = -ab$; this may be done, similarly using the Distributive Rule.